NASA CONTRACTOR REPORT

NASA CR-159

NASA CR-159

(ACCESSION NUMBER) (THRU)

52 (PAGES) 1 (CODE)

(NASA CR OR TMX OR AD NUMBER) 07 (CATEGORY)

FACILITY FORM 602

STUDIES OF ANTENNAS FOR SPACE VEHICLES

GPO PRICE $ ______

OTS PRICE(S) $ ______

Hard copy (HC) 3.00

Microfiche (MF) .50

by P. L. E. Uslenghi

Prepared under Grant No. NsG-444 *by*
UNIVERSITY OF MICHIGAN
Ann Arbor, Mich.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1965

NASA CR-159

STUDIES OF ANTENNAS FOR SPACE VEHICLES

By P. L. E. Uslenghi

Prepared under Grant No. NsG-444 by
UNIVERSITY OF MICHIGAN
Ann Arbor, Mich.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Office of Technical Services, Department of Commerce,
Washington, D.C. 20230 -- Price $3.00

ABSTRACT

1-7451

Various problems associated with space vehicle microwave antennas are discussed. Analytical studies of relationships between far field radiation patterns and near field current distributions and surface configurations, of the effect of the vehicle on antenna performance and of the optimization of preassigned radiation patterns are carried out. An experimental investigation on omnidirectional antennas is described. Other related problems such as a geometrical study of dielectric lenses and high frequency scattering from coated bodies are also considered.

TABLE OF CONTENTS

I
INTRODUCTION

The major demand on communication and telemetering antennas installed on space vehicles is that they provide pattern coverage sufficient to illuminate specified ground stations for all attitudes and trajectories to be experienced by the vehicle. For a spherical spacecraft, a completely omnidirectional radiation pattern is desirable. For cylindrical bodies that may spin but have attitude control, a pattern omnidirectional about the axis of the vehicle and extending fore and aft to the maximum extent possible is required. For space vehicles with more complete control of attitude and aspect, more directional antennas would be adequate.

In the past few years, it has frequently been necessary to meet this demand with techniques and equipment already available owing to the urgency in space programs. However, the necessity was felt for more systematic investigations.

Sixteen months ago, the Radiation Laboratory of The University of Michigan initiated a theoretical and experimental research program to improve the performance of antennas for aerospace vehicles, under NASA Research Grant NsG-444. The two major investigations to be carried out under this program were the reconstruction of the near field from a given far field pattern by analytical and by computer methods, and the study of the effect of vehicle size and shape on the performance of the antenna. The entire program was estimated to require about three years of effort. The research developed during the first sixteen months is outlined below, and will be described in detail in the following sections.

In many cases the space vehicle is an elongated body of revolution, and the microwave antennas are located in the central portion of the body. The creation of a good fore-and-aft coverage is then relatively easy to achieve and requires, for instance, only a radiator of axial dimension small in comparison with the wavelength The realization of a nearly omnidirectional pattern in a plane perpendicular to the axis of symmetry of the body presents difficulties whenever the radius of the surface is not small compared to the wavelength, and therefore the efforts were mainly directed toward the synthesis of this azimuthal pattern. In this research, it was assumed for simplicity that the body is a circular cylinder of infinite length; this assumption is justified because for a long body with a central cylindrical part on which the antenna is located, the two end parts play a minor role in shaping the pattern.

In order to achieve a sensibly uniform coverage in the azimuthal plane other than with arrays of slots, various schemes for providing a distributed excitation have been examined. One of the promising schemes was a circumferential wire fed from within the cylinder. The experimental results were reasonably encouraging, and are described in detail in Section 2.

The problem of the reconstruction of the near field from a given far field pattern by analytical methods has been thoroughly investigated. A variety of pattern optimization procedures for one or more radiating slots on a cylinder have been analyzed, and two of the results obtained deserve particular attention. Firstly, the best mean-squared approximation to the prescribed radiation pattern of a circumferential slot of arbitrary width on a perfectly conducting cylinder of arbitrary radius has been determined for the case in which the aperture distribution function is a trigonometric polynomial of preassigned degree. Secondly, the case of one or two slots on an imperfectly conducting cylinder of large radius was considered, and a sequential method of optimization was arrived at which is well suited to computation. Since in many instances only the power pattern is specified, it is of interest to know the extent to which the field on the vehicle's surface is thereby determined; this problem has been solved for the special cases of an infinite circular cylinder and of a sphere. A more detailed description of the above-mentioned theoretical analyses may be found in Section 3.

The desired coverage in the fore-and-aft direction of a space vehicle is neither as important nor as difficult as the realization of a particular azimuthal pattern. However, as a contribution to the former problem the effect of a surface "kink" (such as at the join of a cone and a cylinder) has been analyzed. Detailed of this investigation are given in Section 4.

The electromagnetic field produced by the antenna of a space vehicle can be rigorously determined only for a few simple geometrical configurations of the system antenna-spacecraft. In a paper which has recently been completed (and submitted for publication in Applied Scientific Research), the exact diffracted field has been determined for the case in which the antenna is an axially oriented electric dipole located on the axis of symmetry of a perfectly conducting conical ring. This research is described in Section 5.

As by-products of the studies of various schemes for providing a distributed excitation on the surface of the spacecraft, two new analyses of scattering from a coated cylinder (accepted for publication in the Canadian Journal of Physics) and from a coated sphere (submitted to Alta Frequenza) were completed. The possibility of obtaining omnidirectionality by surrounding the source (or sources) with a medium composed of various layers with variable refractive indices has also been considered and an initial study of the problem led to two papers dealing with dielectric lenses. The first of these has been accepted for publication in the Canadian Journal of Physics and the second was presented at the URSI-IEEE Fall Meeting in Urbana, Illinois on October 13, 1964. Summaries of these papers are included in Section 6.

The efforts which were given the analytical methods for reconstructing the near field from a given far field pattern have led to new results, which will prove very helpful in reconstructing the near field by machine methods during the future phases of the program. It was in fact felt that such analytical investigations were a necessary prerequisite for the development of feasible computational programs for explicitly carrying out the reconstruction of the fields at the surface of a given spacecraft, for a preassigned radiation pattern.

Many details on the theoretical analyses and experimental investigations carried out under Grant NsG-444 are to be found in the original memoranda and papers, which are listed as references at the end of this report. In the following sections the rationalized MKS system of units is used and the time-dependence factor $e^{-i\omega t}$ is omitted throughout.

II
EXPERIMENTAL INVESTIGATION ON OMNIDIRECTIONAL ANTENNAS

The goal of the experimental study described in this section is to build a new type of microwave spacecraft antenna satisfying the following requirements:

(i) The radiation pattern is omnidirectional in a plane perpendicular to the axis of the space vehicle and has a beam as wide as possible in the fore-aft direction, for a diameter of the vehicle cross section of the order of, or greater than, a few wavelengths.

(ii) The antenna is broadband (e.g. in the S- and X-bands).

(iii) The antenna is incorporated in the vehicle, without requiring movable mechanical parts (like extensible masts) which would diminish its reliability, and without decreasing the mechanical strength of the spacecraft.

(iv) The antenna structure occupies a small portion of the surface of the vehicle.

(v) The antenna feeding system occupies a small portion of the available volume inside the spacecraft.

In the past, most attempts to realize an omnidirectional azimuthal pattern have employed either an antenna located outside the vehicle on its axis of revolution (e.g. a biconical antenna) or a series of conventional radiators located on the surface of the spacecraft and equally spaced around its circumference. In the first case, an extensible mast is usually employed to separate the antenna from the vehicle, thus avoiding undesired modifications to the shape of the pattern in the fore-aft direction. In this way, the reliability of the system is diminished and the dynamical stability of the rotation about the axis of symmetry may be compromised.

In the flush-mounted case where slots are used, the feeding system is complicated, uneconomical as regards space, the structure of the spacecraft is weakened, the frequency independence is not achieved, and the system can be used only if the diameter of the vehicle cross section is sufficiently large with respect to the wavelength. Finally, all multi-radiator systems lead to a pattern with as many minima as there are radiators, and the avoiding of (deep) minima is essential.

In order to overcome all these difficulties, the launching of a circumferential surface wave whose radiation can be controlled as it progresses around the cylindrical vehicle was attempted. The launching device consists of a single wire which lies in the equatorial plane of the spacecraft, and which encircles the vehicle in the circumferential direction running at a suitably chosen distance above its surface. The wire is attached to the center conductor of a feeding coaxial line located inside the vehicle through a hole in the vehicle surface.

The electromagnetic energy propagates along the transmission line consisting of the metallic surface and the wire above it and is radiated into space as it creeps around the vehicle. The various geometrical parameters involved can be used to control both the launching of the surface wave and its radiation. The number of parameters at our disposal may be increased, e.g. by using a dielectric strip located either above the wire or on the metallic surface below it, or two parallel dielectric strips lying on the surface with the wire between them.

The proposed scheme obviously satisfies the requirements (iii), (iv) and (v). The various parameters involved are then to be chosen in such a way that also the requirements (i) and (ii) are met. In order that the omnidirectionality may be achieved, it is necessary to avoid the presence of standing waves around the cylinder, which would originate maxima and minima in the radiation pattern. In order to achieve frequency independence, the wire must encircle the cylinder for no more than a complete turn, and the surface wave must be completely eliminated after one turn. On the other hand, if one wants an omnidirectional pattern, the surface wave must still be present at the end of the first turn around the cylinder, and must then be either radiated or absorbed. It was established (Uslenghi, 1964a) that the surface wave must be absorbed by means of one or more layers of absorbers, whose optimum shape and dimensions are to be determined by experiment, since a theoretical investigation of this problem is too complicated. The absorber must be tapered at both ends in order to avoid sensible reflections from both the wave launched along the wire and that launched from the feeding point in the opposite direction.

In conclusion, it appears that in order to obtain good experimental results, one must choose a system of the type shown in Fig. 1. The inner conductor of the

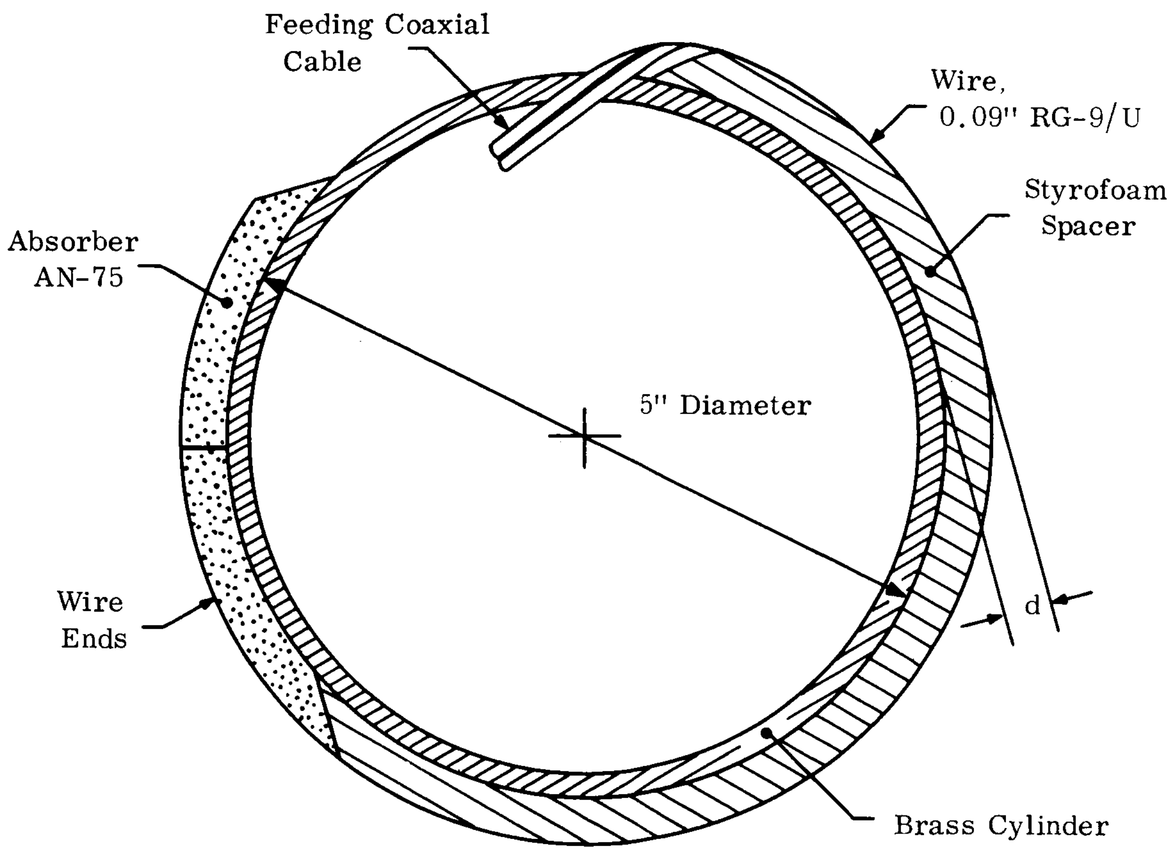

FIGURE 1

feeding coaxial line is gradually brought to a certain distance from the cylinder surface, then it runs around the cylinder for more than 180^{o}. The spacing between wire and cylinder is maintained by a styrofoam strip. It was found experimentally that the most satisfactory termination in view of the behavior of the radiation pattern consists in "sinking" the wire into the absorber (AN-75, in the specific case) without putting it in electrical contact with the cylinder surface. The cylinder was made of brass, it was about four feet long and with an external diameter of five inches. The wire (0.09" in diameter) was located at the same distance (two feet) from both ends of the cylinder.

The experiments consisted of measuring the radiation pattern and the field intensity in the vicinity of the wire, for frequencies varying from 7 to 11 Gc, and for the following values of the spacing d between wire and cylinder: 5/16", 7/16", 9/16", 11/16", 13/16". As a consequence of the fact that the distance of the probe from the wire was varying while the cylinder was rotating about its axis, only qualitative information can be obtained from the diagrams of the field intensity near the wire as a function of the angular distance along the wire. Not surprisingly, the current in the wire steadily decreases as one goes farther and farther from the feeding point.

The diagram of the radiated power as a function of the angular distance from the feeding point presents an irregular behavior at first, then it rapidly increases as the surface wave builds up. It remains approximately constant (power oscillations confined within 2 db) over a certain angular range, then it decreases with larger and larger oscillations. These oscillations have their maximum amplitudes around 270^{o} from the feeding point. All the experimental efforts to eliminate them have been fruitless.

Similar results were obtained for the case of an aluminum cylinder with an 18" diameter. However, in this case it was necessary to bind the surface wave more closely to the cylinder surface, and a dielectric strip 4" wide, 1/2" thick and with a relative dielectric constant of either 4 or 6 was therefore inserted between the wire and the cylinder.

In conclusion, it appears that the proposed launching scheme can produce a nearly omnidirectional pattern (power oscillations of less than 2 db) over an angular range of 100^o-120^o.

In relation to the experimental work just described, it was of interest to determine the attenuation by radiation of a circumferential TM surface wave creeping around a dielectrically coated cylinder. The analysis was simplified by considering a coated plane and by showing that an impedance boundary condition holds on the outer surface of the coating layer. The relative surface impedance Z is given by:

$$Z = -i\sqrt{\epsilon - 1 - \frac{\gamma_1^2}{k^2}} \ , \tag{1}$$

where γ_1 satisfies the transcendental equation

$$\gamma_1^2 \left(1 + \frac{1}{\epsilon^2} \tan^2 \gamma_1 d\right) = k^2(\epsilon - 1) \ , \tag{2}$$

$k = 2\pi/\lambda$ is the free space wave number, and ϵ and d are the relative dielectric constant and the thickness of the coating layer. In order that only the fundamental TM surface wave may propagate, it is necessary that

$$\begin{aligned} \frac{d}{\lambda} &< \frac{1}{2\sqrt{\epsilon - 1}} \ , && \text{if } \epsilon \leqslant 2.25, \\ &< \frac{1}{2\pi\sqrt{\epsilon - 2}} \arctan \frac{\epsilon}{\sqrt{\epsilon - 2}} \ , && \text{if } \epsilon \geqslant 2.25 \ . \end{aligned} \tag{3}$$

The power attenuation of a surface wave creeping around a coated cylinder of outer radius a is given by:

$$\text{power attenuation (db/m)} = 8.68\, k\, \mathrm{Im}\left(\sqrt{1 - Z^2}\right), \quad (k \text{ in } m^{-1}), \tag{4}$$

where $Z = R - iX$ is the relative surface impedance. Then, if $a \gg \lambda$,

$$X \sim \sqrt{\epsilon - 1 - \frac{\gamma_1^2}{k^2}} \, . \tag{5}$$

An approximate evaluation of R has proved that the power attenuation is of the order of 4 db/radian for $ka \gg 1$, thus confirming the result that Barlow (1959) had previously found for $ka \sim 200$.

III
SYNTHESIS OF RADIATION PATTERNS

Let us consider an infinitely long circular cylinder of radius a at the surface of which the longitudinal component of the electric field is $A(\phi')$, $-\pi \leqslant \phi' \leqslant \pi$. The field pattern $P(\phi)$ (coefficient of $e^{ikr}/\sqrt{kr}$ in the expression that gives the radiated electric field at a large distance r from the cylinder axis) is then:

$$P(\phi) = \sqrt{\frac{2}{\pi i}} \cdot \sum_{n=-\infty}^{\infty} \frac{e^{in(\phi - \frac{\pi}{2})}}{H_n^{(1)}(ka)} a_n, \tag{6}$$

where

$$a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} A(\phi') e^{-in\phi'} d\phi' , \tag{7}$$

and ϕ and ϕ' are measured from the same half-plane. By expanding $P(\phi)$ and $A(\phi')$ in Fourier series

$$P(\phi) = \sum_{n=-\infty}^{\infty} p_n e^{in\phi}, \qquad A(\phi') = \sum_{n=-\infty}^{\infty} a_n e^{in\phi'} , \tag{8}$$

where

$$p_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} P(\phi) e^{-in\phi} d\phi \tag{9}$$

and a_n is given by (7), one sees that since relation (6) must hold for all $-\pi \leqslant \phi \leqslant \pi$,

$$a_n = \sqrt{\frac{\pi i}{2}}\; i^n H_n^{(1)}(ka)\, p_n , \tag{10}$$

and therefore the nth Fourier coefficient of $A(\phi')$ is uniquely determined by the nth Fourier coefficient of $P(\phi)$, and vice versa.

If only the power polar diagram (or the power distribution on the surface) is known, the surface field is no longer uniquely determined. In general there is now an infinity of surface distributions which will generate a given far field power pattern. Thus, for example, from the power pattern $|P(\phi)|^2$ we have that

$$|P(\phi)|^2 = \sum_{s=-\infty}^{\infty} \beta_s e^{is\phi} , \tag{11}$$

where

$$\beta_s = \sum_{t=-\infty}^{\infty} p_t p_{t-s}^* \tag{12}$$

and the asterisk denotes complex conjugate. Recalling that the surface coefficients a_n are related to the pattern coefficients p_n through equation (10), it is clear that the surface field is determined only to the extent that the p_n's are obtainable from the β_s's using (12).

A similar analysis may be performed for a sphere. Confining our attention to the scalar case for simplicity, the azimuthal polar diagram can be written as

$$P(\phi) = \sum_{n=0}^{\infty} p_n P_n(\cos\phi) , \tag{13}$$

where the p_n's are constants and $P_n(\cos\phi)$ is the Legendre function of zero order. The power radiation pattern is then given by (Senior, 1964)

$$|P(\phi)|^2 = \sum_{s=0}^{\infty} \left\{\beta_{2s} P_{2s}(\cos\phi) + \beta_{2s+1} P_{2s+1}(\cos\phi)\right\} , \tag{14}$$

where:

$$\beta_{2s} = \left\{ \sum_{n=0}^{\infty}{}'' \sum_{m=|n-2s|}^{n+2s}{}'' + \sum_{n=0}^{\infty}{}' \sum_{m=|n-2s|}^{n+2s}{}' \right\} p_n p_m^* \gamma_{s+1/2(m-n)} (n, m) , \qquad (15)$$

$$\beta_{2s+1} = \left\{ \sum_{n=0}^{\infty}{}' \sum_{m=|n-1-2s|}^{n+1+2s}{}'' + \sum_{n=0}^{\infty}{}'' \sum_{m=|n-1-2s|}^{n+1+2s}{}' \right\} p_n p_m^* \gamma_{s+1/2(m-n+1)} (n, m) , \qquad (16)$$

the single (double) prime denotes summation over odd (even) values, and

$$\gamma_s(n, m) = (s + \frac{1}{2}) \int_{-1}^{+1} P_n(\mu) P_m(\mu) P_s(\mu) d\mu. \qquad (17)$$

Each coefficient β_s is therefore the sum of two products of a finite and an infinite sum. Recalling that the coefficients a_n in the Legendre function expansion of the surface field are trivially related to the p_n's, one sees that a knowledge of the power polar diagram determines the surface field only to the extent that the p_n's are determined by the β_s's using equations (15) and (16). It is also apparent that there is no hope of obtaining any simple specification of the freedom left in the choice of the p_n's.

In most practical applications, only the amplitude of the far field radiation pattern is given. The phase of the pattern should then be chosen in such a way as to reproduce the given amplitude pattern within a preassigned maximum mean-squared error by means of the simplest surface field possible. This approach is the most logical one but, as it appears from the previous considerations, it is not without difficulties, and therefore the far field pattern is ordinarily taken to be real for all $-\pi \leqslant \phi \leqslant \pi$. The latter approach will now be followed in determining the best mean-squared approximation to the prescribed amplitude radiation pattern of the far field originated by a circumferential slot of arbitrary width (and infinite axial length) on a perfectly conducting cylinder of arbitrary radius a.

The optimization will be carried out for the simple case in which the aperture distribution function $A_N(\phi')$ is a trigonometric polynomial of preassigned degree N. Three cases are considered in the following:

(i) Annular slot; $P(\phi)$ assigned in $|\phi| \leqslant \pi$.

(ii) Annular slot; $P(\phi)$ assigned in $-\pi \leqslant \beta_o - \Delta \leqslant \phi \leqslant \beta_o + \Delta \leqslant \pi$.

(iii) Slot of angular width 2α, centered at $\phi' = 0$; $A(\phi')$ is required to be zero for $0 < \alpha < |\phi'| \leqslant \pi$, and $P(\phi)$ is assigned in $|\phi| \leqslant \pi$.

Case (i)

If the order of harmonics in $A(\phi')$ is limited to a maximum N, then both the aperture function and the pattern are trigonometric polynomials of degree N. The optimization problem may then be phrased as follows: what are the coefficients p_n that give the best mean square fit of the polynomial

$$P_N(\phi) = \sum_{n=-N}^{N} p_n e^{in\phi} \tag{18}$$

to the prescribed pattern $P(\phi)$? The answer is well known: the p_n's are the Fourier coeffic'ents given by (9), where now $n = 0, \pm 1, \ldots, \pm N$. The optimum coefficients a_n of the aperture function

$$A_N(\phi') = \sum_{n=-N}^{N} a_n e^{in\phi'} \tag{19}$$

are then easily obtained through relation (10).

Case (ii)

The mean-squared error ϵ_N between the approximation $P_N(\phi)$ given by (18) and the prescribed pattern $P(\phi)$ is:

$$\epsilon_N = \int_{\beta_o - \Delta}^{\beta_o + \Delta} \left| P_N(\phi) - P(\phi) \right|^2 d\phi. \tag{20}$$

The (2N+1) unknowns p_n are given by the (2N+1) linear algebraic equations:

$$\sum_{m=-N}^{N} \frac{\sin(m-n)\Delta}{(m-n)\Delta} p_m e^{i(m-n)\beta_o} = \frac{1}{2\Delta} \int_{\beta_o-\Delta}^{\beta_o+\Delta} P(\phi) e^{-in\phi} d\phi, \quad (n = 0, \pm 1, \ldots \pm N). \tag{21}$$

Once the optimum p_n's are derived by means of (21), the optimum a_n's are easily computed through (10).

Case (iii)

The aperture function $A(\phi')$ is now given by the relation:

$$A(\phi') = \sum_{n=-N}^{N} \gamma_m e^{i \frac{m\pi}{\alpha} \phi'} , \tag{22}$$

which substituted into (6) yields:

$$P_N(\phi) = \frac{\alpha}{\pi} \sqrt{\frac{2}{\pi i}} \sum_{m=-N}^{N} \gamma_m \sum_{n=-\infty}^{\infty} \frac{\sin(m\pi - n\alpha)}{(m\pi - n\alpha)} \frac{e^{in(\phi - \frac{\pi}{2})}}{H_n^{(1)}(ka)} . \tag{23}$$

The (2N+1) coefficients γ_m are determined by the necessary conditions for a minimum of the mean-squared error ϵ_N, i.e. by the (2N+1) linear algebraic equations:

$$\sum_{m=-N}^{N} \gamma_m \sum_{n=-\infty}^{\infty} \frac{1}{H_n^{(1)}(ka) H_n^{(2)}(ka)} \frac{\sin(\ell\pi - n\alpha)}{\ell\pi - n\alpha} \frac{\sin(m\pi - n\alpha)}{m\pi - n\alpha} =$$

$$\frac{\pi}{\alpha} \sqrt{\frac{\pi i}{2}} \sum_{n=-\infty}^{\infty} \frac{i^n}{H_n^{(2)}(ka)} \frac{\sin(\ell\pi - n\alpha)}{\ell\pi - n\alpha} p_n, \quad (\ell = 0, \pm 1, \ldots, \pm N) \tag{24}$$

where the p_n's are given by (9). The problem of evaluating the infinite series which appear in equations (24) will be dealt with during the future phases of the program, that will include extensive computations of the mean-squared error ϵ_N as a function of N, ka and α, for at least the case in which $P(\phi)$ is independent of ϕ (omnidirectionality).

The previous choice of a trigonometric polynomial as aperture distribution function is justified by the simplicity of the optimization procedure, but it is not necessarily the most logical one. For instance, it would be interesting to optimize the pattern for a preassigned maximum value of the antenna quality factor Q. However, the analogy with the complex angles of radiation which, for a plane slot, are usually regarded as the depository of the reactive power and which enabled Rhodes (1963) to optimize a given pattern for a preassigned value of the superdirective ratio, is not readily apparent. In other words, further study is required to arrive at a satisfactory definition of the antenna quality factor for a radiating slot on a cylindrical body. Moreover, a treatment analogous to that of Rhodes would probably involve functions which are complete and orthogonal over two finite ranges; substantial progress in the determination of these functions has been made recently, and the study continues.

Rhodes' method is the only known technique for obtaining the optimum approximation to a given pattern from a rational viewpoint, i.e. taking into account the practical limitations on the antenna quality. Unfortunately, it is applicable only when pattern P and aperture distribution A are related through a finite Fourier transform.

$$P(u) = \int_{-1}^{+1} A(v)\, e^{-iuv}\, dv\ , \qquad A(v) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P(u)\, e^{iuv}\, du\ . \tag{25}$$

However, if the relationship between P and A is sufficiently close to that given by (25), one can still use Rhodes' method. Specifically, let us assume that instead of the first of (25), the following relation holds:

$$P(u) = \int_{-1}^{+1} A(v) \left\{1 + \sigma(u, v)\right\} e^{-iuv} dv, \tag{26}$$

then Rhodes' method can be applied to each one of the successive approximations:

$$P(u) - \int_{-1}^{+1} A_{n-1}(v)\, \sigma(u, v)\, e^{-iuv} dv = \int_{-1}^{+1} A_n(v)\, e^{-iuv} dv, \quad (n = 1, 2, \ldots), \tag{27}$$

$$A_o(v) = 0 \;, \tag{28}$$

where $A_n(v)$ should converge to $A(v)$, provided that $|\sigma(u, v)|$ is sufficiently small compared to unity. This iteration procedure has been applied to a particular case, and the results of the investigation are given in the following.

Consider an infinite circular cylinder of radius a and assume that on its surface an impedance boundary condition holds. The relative surface impedance Z may account, for instance, for the roughness of the surface, or for the presence of coating layers. Two radiating strip sources of the same width b are located at $\phi = 0$ and $\phi = \pi$ on the surface of the cylinder, as illustrated in Fig. 2. Their aperture distribution functions $A(x)$, $|x| \leqslant b/2$, and $\tilde{A}(\tilde{x})$, $|\tilde{x}| \leqslant b/2$, are to be chosen in such a way that the best mean-squared approximation to the azimuthal radiation patterns $P(\phi)$, $|\phi| \leqslant \pi/2$, and $\tilde{P}(\phi)$, $\pi/2 < |\phi| \leqslant \pi$, is achieved, for a preassigned value of Taylor's superdirective ratio.

Suppose that $\mathrm{Re}\, Z \geqslant 0$ and $\mathrm{Im}\, Z < 0$ so that a surface wave can propagate around the cylinder, and assume that the creeping waves may be neglected. If

$$b/2a \ll 1, \qquad \lambda/a \ll 1, \qquad \exp(-2\pi a\, \mathrm{Im}\, k_1) \ll 1, \tag{29}$$

where

$$k_1 = k\sqrt{1 - Z^2} \;, \tag{30}$$

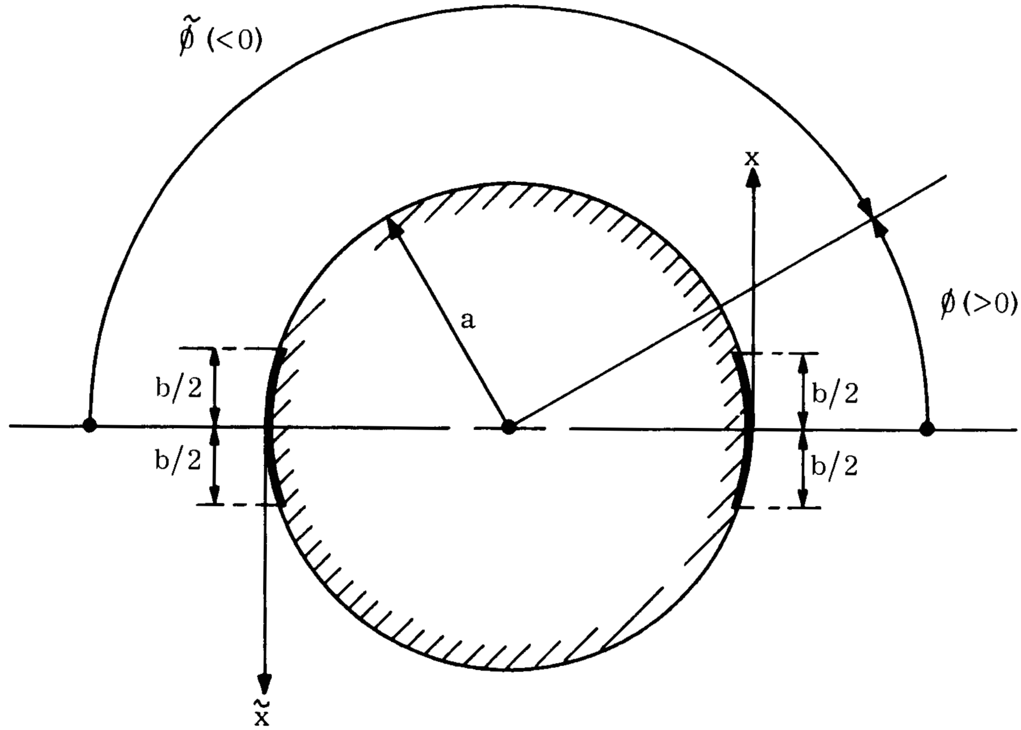

FIGURE 2

then the radiation field patterns $P(\phi)$ and $\tilde{P}(\phi)$ are given by (Uslenghi, 1963):

$$P(\phi) = \int_{-b/2}^{b/2} A(x) \left[e^{-ikx\sin\phi} + M e^{ika\cos\phi} \cos\left\{ k_1 a \left(\left| \phi - \frac{x}{a} \right| - \pi \right) \right\} \right] dx +$$

$$M e^{ika\cos\phi} \int_{-b/2}^{b/2} \tilde{A}(\tilde{x}) \cos\left\{ k_1 a \left(\left| \tilde{\phi} - \frac{\tilde{x}}{a} \right| - \pi \right) \right\} d\tilde{x}, \quad \left(|\phi| \leqslant \frac{\pi}{2} \right), \qquad (31)$$

$$\tilde{P}(\phi) = \int_{-b/2}^{b/2} \tilde{A}(\tilde{x}) \left[e^{-ik\tilde{x}\sin\tilde{\phi}} + M e^{-ika\cos\phi} \cos\left\{ k_1 a \left(\left| \tilde{\phi} - \frac{\tilde{x}}{a} \right| - \pi \right) \right\} d\tilde{x} + \right.$$

$$+ M e^{-ika\cos\phi} \int_{-b/2}^{b/2} A(x) \cos\left\{ k_1 a \left(\left| \phi - \frac{x}{a} \right| - \pi \right) \right\} dx, \quad (\pi/2 < |\phi| \leqslant \pi), \tag{32}$$

where

$$\tilde{\phi} = \phi - \pi \ , \qquad \text{for } 0 \leqslant \phi \leqslant \pi \ ,$$

$$= \phi + \pi \ , \qquad \text{for } -\pi \leqslant \phi \leqslant 0 \ , \tag{33}$$

$$M = -i\sqrt{32\pi ka} \ \frac{Z^{3/2}}{\sqrt{1-Z^2}} \ \exp\left\{ i3Zka + ik_1 a \left[\pi + \sin^{-1}\left(\sqrt{1-Z^2} \right) - 2\cos^{-1}(-Z) \right] \right\} \tag{34}$$

Given P, $\tilde{P}$ and the superdirective ratio, one can compute Rhodes' optimum distributions A and $\tilde{A}$ by successive approximations. The recurrence formulas are:

$$P(\phi) - MQ_n(\phi)\, e^{ika\cos\phi} = \int_{-b/2}^{b/2} A_n(x)\, e^{-ikx\sin\phi}\, dx \ , \quad (|\phi| \leqslant \pi/2), \tag{35}$$

$$\tilde{P}(\phi) - MQ_n(\phi)\, e^{-ika\cos\phi} = \int_{-b/2}^{b/2} \tilde{A}_n(\tilde{x})\, e^{-ik\tilde{x}\sin\tilde{\phi}}\, d\tilde{x}, \quad (\pi/2 < |\phi| \leqslant \pi), \tag{36}$$

where:

$$n = 1, 2, \ldots,$$

$$Q_n(\phi) = \int_{-b/2}^{b/2} A_{n-1}(x) \cos\left\{k_1 a\left(\left|\phi - \frac{x}{a}\right| - \pi\right)\right\} dx +$$

$$\int_{-b/2}^{b/2} \tilde{A}_{n-1}(\tilde{x}) \cos\left\{k_1 a\left(\left|\tilde{\phi} - \frac{\tilde{x}}{a}\right| - \pi\right)\right\} d\tilde{x}, \tag{37}$$

$$A_o(x) = \tilde{A}_o(\tilde{x}) = 0 \ . \tag{38}$$

The calculations involved in this optimization procedure are easily performed with the aid of a computer.

IV
THE EFFECT OF A SINGULARITY IN THE VEHICLE SHAPE

In this section, the diffraction of a plane electromagnetic wave incident on a perfectly conducting infinite cylinder and polarized in a plane parallel to the cylinder axis is considered. The exact solution of this two-dimensional problem is known only for a few simple shapes of the cylinder cross section, like the circle or the ellipse. For more general shapes, various approximation methods have been developed. In the high frequency limit, all these methods except Garabedian's (1955) require a certain degree of smoothness of the boundary (usually, the continuity of the curvature). The special cases of a wedge-type singularity and of a discontinuity in the curvature were considered by Keller (1962) and Weston (1962) respectively.

Certain types of singularities of the boundary can be handled by mapping that region of the plane which is external to the cylinder cross section into another region with a geometrically simpler boundary. Such a conformal transformation preserves the right angle between the direction of propagation of the wave and the wavefront. Moreover, the order of the singularity of the mapping function on the boundary is proportional to that of the geometry of the boundary itself (Warshawski, 1935).

In the following, the study of certain irregularities of the boundary, such as discontinuities in curvature and higher order derivatives or small, periodic, smooth corrugations, is performed by conformal mapping method. Firstly, an integral equation governing the scattered field is formulated. In order to solve this equation, it is then assumed that both original and transformed boundary curves have continuous tangent lines and that the distance between the two curves is everywhere small compared to the wavelength.

Consider the perfectly conducting infinite cylinder whose cross section is shown in Fig. 3, and let $E_{\zeta} = e^{ik\xi} = -\sqrt{\mu_o/\epsilon_o}\, H_{\eta}$ be a plane incident electromagnetic wave which generates the scattered field $E_{\zeta} = \tilde{u}(w)$, where $w = \xi + j\eta = \rho e^{j\phi}$. The

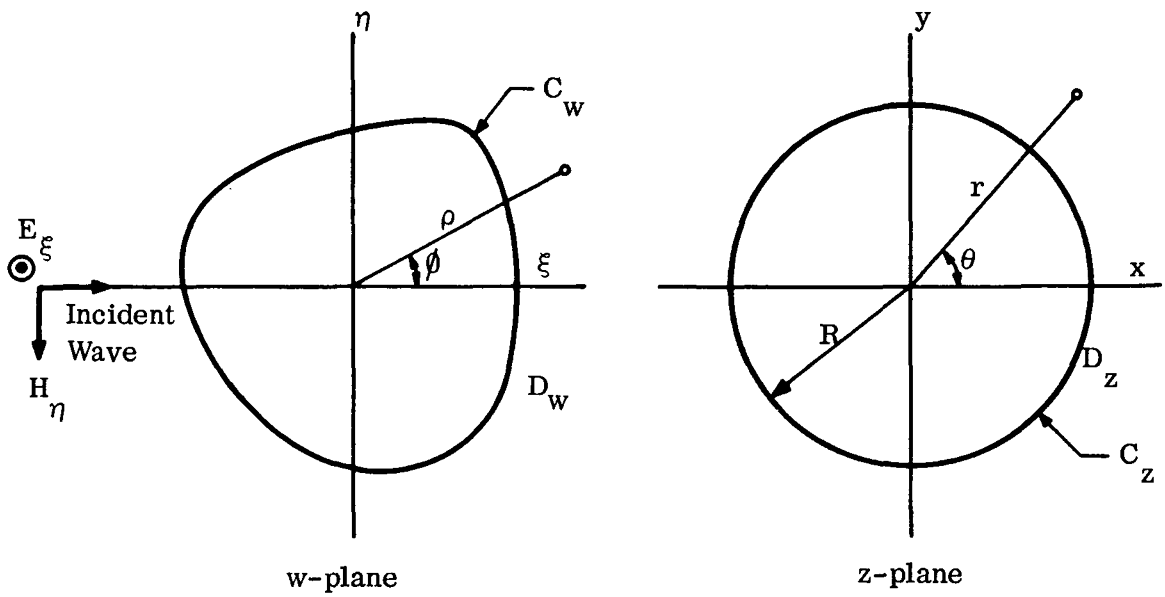

FIGURE 3

domain D_w of the w-plane is conformally mapped into the domain D_z of the z-plane ($z = x + jy = re^{j\theta}$) by means of a function $w = F(z)$, that transforms the boundary C_w of D_w into the circumference C_z of radius R. The mapping function F(z), which is analytic in D_z and well defined on C_z, can be expanded as follows:

$$F(z) = z + \tilde{g}(z) = z + \sum_{n=1}^{\infty} A_n / z^n \, , \qquad \text{for } |z| > R. \tag{39}$$

The transformed scattered field

$$u(z) = \tilde{u}(w)\Big|_{w=F(z)} \tag{40}$$

must satisfy the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 |F'(z)|^2 u = 0 \qquad \text{in } D_z, \tag{41}$$

the boundary condition

$$u = -e^{ikRe\, F(z)} \qquad \text{on} \qquad C_z, \tag{42}$$

and the radiation condition

$$\lim_{r \to \infty} r^{1/2} \left(\frac{\partial u}{\partial r} - iku \right) = 0. \tag{43}$$

The solution of this boundary value problem can be reduced to the solution of the following Fredholm integral equation of the second kind (see also Garabedian, 1955):

$$u(\vec{r}_1) = u_1(\vec{r}_1) + k^2 \iint\limits_{|z|>R} \left\{ |F'(z)|^2 - 1 \right\} G(\vec{r}, \vec{r}_1)\, u(\vec{r})\, dxdy\ , \tag{44}$$

where

$$u_1(\vec{r}_1) = \oint\limits_{|z|=R} e^{ikRe\, F(z)} \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial r} R\, d\theta \tag{45}$$

is a known quantity, and Green's function G is given by

$$G(\vec{r}, \vec{r}_1) = \frac{i}{4} \sum_{n=-\infty}^{\infty} H_n^{(1)}(kr_1) \left\{ J_n(kr) - \frac{J_n(kR)}{H_n^{(1)}(kR)} H_n^{(1)}(kr) \right\} e^{in(\theta_1 - \theta)}, \tag{46}$$

for $r_1 > r$.

It can be proven that the kernel of equation (44) is bounded everywhere except at $\vec{r} = \vec{r}_1$, where G has a logarithmic singularity.

Let the equation $\rho = \rho(\phi)$ represent the boundary curve C_w, and suppose that the relation

$$|\rho(\phi) - R| \leqslant \delta \ , \tag{47}$$

where δ is a positive constant, is valid for all $-\pi < \phi \leqslant \pi$. If $k\delta \ll 1$, and if one neglects terms $O\left[(k\delta)^2\right]$, then the solution of equation (44) is:

$$u(\vec{r}_1) = \oint_{|z|=R} \left\{1 + ik\dot{R}e\tilde{g}(z)\right\} e^{ikR\cos\theta} \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial r} R\, d\theta + $$
$$+ 2k^2 \iint_{|z|>R} \mathrm{Re}\left[\tilde{g}'(z)\right] G(\vec{r}, \vec{r}_1)\, u_o(\vec{r})\, dxdy + O\left[(k\delta)^2\right] , \tag{48}$$

where

$$u_o(\vec{r}) = \oint_{|z|=R} e^{ikR\cos\theta'} \frac{\partial G(\vec{r}, \vec{r}')}{\partial r'} R d\theta'$$

$$= - \sum_{n=-\infty}^{\infty} i^n \frac{J_n(kR)}{H_n^{(1)}(kR)} H_n^{(1)}(kr)\, e^{in\theta} . \tag{49}$$

Since $F(z) \sim z$ as $|z| \to \infty$, the far scattered field $\tilde{u}(\vec{\rho}_1)$ is immediately known, once $u(\vec{r}_1)$ has been determined.

As an example, let us consider the case in which the boundary curve C_w is given by the relation

$$\rho(\phi) = R+p(\phi) = R+\sum_{n=1}^{\infty}(a_n \cos n\phi + b_n \sin n\phi), \tag{50}$$

where $p(\phi)$ is bounded, piecewise continuous, and $\max|p(\phi)| = \delta$. Then the mapping function $F(z)$ is:

$$F(z) = z + \sum_{n=1}^{\infty} \frac{a_n + ib_n}{(z/R)^{n-1}} + O(\delta^2) . \tag{51}$$

In particular, if

$$\rho(\phi) = R+a_{\ell} \cos \ell\phi + b_{\ell} \sin \ell\phi \tag{52}$$

with $k\sqrt{a_{\ell}^2+b_{\ell}^2} \ll 1$, then formula (48) holds, and one finds that the far scattered field $\tilde{u}(\vec{\rho}_1)$ is given by:

$$\tilde{u}(\vec{\rho}_1) = \tilde{u}_o(\vec{\rho}_1)+\tilde{u}_p(\vec{\rho}_1) , \tag{53}$$

where $\tilde{u}_o(\vec{\rho}_1)$ is the field scattered by a circular cylinder of radius R, and

$$\tilde{u}_p(\vec{\rho}_1) \sim \frac{1}{\pi}\sqrt{\frac{a}{\pi k\rho_1}}\, e^{ik\rho_1 - i\frac{3\pi}{4}} \sum_{n=-\infty}^{\infty} \frac{e^{in\phi_1}}{H_n^{(1)}(kR)} \left\{ i^{\ell} \frac{a_{\ell}+ib_{\ell}}{RH_{n+\ell}^{(1)}(kR)} + i^{-\ell} \frac{a_{\ell}-ib_{\ell}}{RH_{n-\ell}^{(1)}(kR)} \right\} \tag{54}$$

is the perturbation due to the term $(a_{\ell} \cos \ell\phi + b_{\ell} \sin \ell\phi)$ in relation (52). The result (54) has been previously obtained by an entirely different method (Clemmow and Weston, 1961), and therefore represents a verification of the validity of solution (48).

The remaining part of this section deals with the effect that the two discontinuities in the curvature of the boundary curve C_w obtained by smoothly joining together a semicircle and a semi-ellipse have on the scattered field. Let a be the radius of the semicircle $\phi_d \leqslant \phi \leqslant \pi + \phi_d$, let 2a be the minor axis and ϵ the eccentricity of the ellipse, and assume that $\pi/2 \leqslant \phi_d \leqslant \pi$ and that the observation point (ρ_1, ϕ_1) is located within the angular region $0 \leqslant \phi_1 \leqslant \pi$. If the eccentricity ϵ is small, so that one may neglect terms $O\left[(kR\,\epsilon^2/2)^2\right]$, then the scattered field is given by (48). We shall indicate with $u_I(\vec{r}_1)$ and $u_{II}(\vec{r}_1)$ respectively the line and the surface integrals in the second member of (48).

An asymptotic evaluation of $u_I(\vec{r}_1)$ in the high frequency limit $(kR \gg 1)$ gives:

$$\tilde{u}_I(\vec{\rho}_1) \sim \sqrt{\frac{2}{\pi k \rho_1}}\; e^{ik\rho_1 - i\frac{\pi}{2}} (\sigma + \sigma^*), \tag{55}$$

where:

$$\sigma = -\frac{i}{2} M^2 \int_{\tilde{V}_{\theta=\pi}}^{-i\infty} e^{-\tilde{V}} f\left[M(\tfrac{\pi}{2} - \theta)\right] \left\{1 + g(-\theta + \pi + \phi_1)\right\} \frac{d\theta}{d\tilde{V}}\, d\tilde{V}\ , \tag{56}$$

$$\sigma^* = \frac{i}{2} M^2 \int_{\tilde{V}^*_{\theta=-\pi}}^{-i\infty} e^{-\tilde{V}^*} f\left[M(\tfrac{\pi}{2} + \theta)\right] \left\{1 + g(-\theta - \pi + \phi_1)\right\} \frac{d\theta}{d\tilde{V}^*}\, d\tilde{V}^*, \tag{57}$$

with

$$\tilde{V} = ikR\left\{\cos(\phi_1 - \theta) - \frac{\pi}{2} + \theta\right\}, \tag{58}$$

$$\tilde{V}^* = ikR\left\{\cos(\phi_1 - \theta) - \frac{\pi}{2} - \theta\right\}, \tag{59}$$

$$f(\mu) = 2i\sqrt{\pi} \sum_{s=1}^{\infty} \frac{e^{i\mu t_s}}{w_1'(t_s)}\ , \qquad (\mu \geqslant 0)\ , \tag{60}$$

$w_1(t)$ is the Airy integral in Fock's notation:

$$w_{1,2}(t) = \sqrt{\pi}\,\left[Bi(t) \pm iAi(t)\right] \quad , \tag{61}$$

t_s is the s-th root of $w_1(t) = 0$, $M = (kR/2)^{1/3}$, and $g(\theta) = ik\,Re\,\tilde{g}(z)$. The line integral (56) (or (57)) has two kinds of branch points: those located at the saddle points $d\tilde{V}/d\theta = 0$ (or $d\tilde{V}^*/d\theta = 0$), and those of $g(\theta)$. The first ones give rise to branch cut integrals which represent that portion u_{Is} of the field

$$\tilde{u}_I = \tilde{u}_{Is} + \tilde{u}_{Id} \tag{62}$$

which is due to specular reflection and to the creeping waves launched at the shadow boundaries $\phi = \pm\pi/2$. The branch cut integrals corresponding to the branch points of $g(\theta)$ represent that contribution $\tilde{u}_{Id}$ to the scattered field $\tilde{u}_I$ which arises from the points of discontinuity in curvature.

Let us consider only the field $\tilde{u}_{Id}(\vec{\rho}_1)$. If the shadow boundary and the points of discontinuity in curvature are far apart, three regions must be separately examined. When $\phi_d - \frac{\pi}{2} < \phi_1 \leqslant \pi$ and $\phi_1 \neq 2\phi_d - \pi$, then

$$\tilde{u}_{Id}(\vec{\rho}_1) \sim \sqrt{\frac{2}{\pi k\rho_1}}\; e^{ik\rho_1 - i\frac{\pi}{4}}\; \sigma_{\theta=\phi_1-\phi_d+\pi} \quad , \tag{63}$$

where

$$\sigma_{\theta=\phi_1-\phi_d+\pi} \cong \frac{4\left(\frac{1}{a} - \frac{1-\epsilon^2}{a}\right)(\cos\phi_d - 2i\sin\phi_d)\cos(\phi_1-\phi_d)}{k\left[4\sin\frac{\phi_1}{2}\cos\left(\frac{\phi_1}{2} - \phi_d\right)\right]^3} \,.$$

$$\cdot \exp\left[-i2kR\sin\frac{\phi_1}{2}\sin\left(\phi_d - \frac{\phi_1}{2}\right)\right] . \tag{64}$$

The second member of (63) represents the field which is scattered by the discontinuity in curvature at $\phi = \phi_d$; the result is very similar to that obtained by Weston(1962) for the other polarization of the incident wave. It can be proven that creeping waves are launched from $\phi = \phi_d$ and $\phi = \phi_d - \pi$, which are $O(k^{-11/6})$ and $O(k^{-13/6})$ respectively.

If $\phi_1 = 2\phi_d - \pi$, then the specular reflection point and the point of discontinuity at $\phi = \phi_d$ coincide and, in first approximation, it is not possible to separate the scattered field due to the discontinuity from the specular reflection term.

When $0 \leqslant \phi_1 \leqslant \phi_d - \frac{\pi}{2}$, one has that

$$\tilde{u}_{Id}(\vec{\rho}_1) \sim \sqrt{\frac{2}{\pi k \rho_1}}\, e^{ik\rho_1 - i\frac{\pi}{4}}\, \sigma_{\theta = \phi_1 - \phi_d + \pi} , \tag{65}$$

where now

$$\sigma_{\theta = \phi_1 - \phi_d + \pi} \cong \frac{\sqrt{\pi}}{2kM} \frac{\left(\frac{1}{a} - \frac{1-\epsilon^2}{a}\right)(2i \sin\phi_d - \cos\phi_d)}{(1-\sin\phi_d)^3} \sum_{s=1}^{\infty} \frac{\exp\left[i(kR + Mt_s)(\phi_d - \frac{\pi}{2} - \phi_1)\right]}{w_1'(t_s)} \tag{66}$$

The contribution (65) represents a creeping wave launched from the discontinuity at $\phi = \phi_d$.

Let us consider the case in which shadow boundary and discontinuity in curvature coincide. If ϕ_1 is bounded away from both zero and π, then $\tilde{u}_{Id}$ is still given by relation (63), where now $\phi_d = \pi/2$. If instead $\phi_1 \sim 0$, then the contribution due to the discontinuity cannot be separated from the field diffracted by the shadow boundary, in first approximation. Finally, if $\phi \sim \pi$, formula (55) holds with the following values of σ and σ^*:

$$\sigma_{\theta=\phi_1-\frac{\pi}{2}} = \frac{i\sqrt{\pi}}{8kM}\left(\frac{1}{a}-\frac{1-\epsilon^2}{a}\right)\sum_{s=1}^{\infty}\frac{\exp\left\{i(kR+Mt_s)(\pi-\phi_1)\right\}}{w_1'(t_s)}, \tag{67}$$

$$\sigma^*_{\theta=\phi_1-\frac{3\pi}{2}} = \frac{i\sqrt{\pi}}{8kM}\left(\frac{1}{a}-\frac{1-\epsilon^2}{a}\right)\sum_{s=1}^{\infty}\frac{\exp\left\{i(kR+Mt_s)(\phi_1-\pi)\right\}}{w_1'(t_s)}. \tag{68}$$

One still has to evaluate the surface integral $u_{II}(\vec{r}_1)$ of relation (48). It is found that

$$\tilde{u}_{II}(\vec{\rho}_1) \sim \sqrt{\frac{2}{\pi k\rho_1}}\, e^{ik\rho_1 - i\frac{\pi}{4}} \sum_{\ell=1}^{\infty} i^{\ell}\,\bar{\sigma}_{\ell}, \tag{69}$$

where

$$\bar{\sigma}_{\ell} = \frac{k}{2}\sum_{n=-\infty}^{\infty}\frac{e^{in\phi_1}}{H_n^{(1)}(kR)}\left[(a_\ell + ib_\ell)\,J_{n+\ell}(kR)\,\frac{H^{(1)}_{n+\ell-1}(kR)}{H^{(1)}_{n+\ell}(kR)} - (-1)^{\ell}(a_\ell - ib_\ell)\,J_{n-\ell}(kR)\,\frac{H^{(1)}_{n-\ell+1}(kR)}{H^{(1)}_{n-\ell}(kR)}\right]. \tag{70}$$

Assuming that $kR \gg 1$ and applying a Watson transformation, one finds that

$$\bar{\sigma}_{\ell} = \bar{\sigma}_{\ell 1} + \bar{\sigma}_{\ell 2}, \tag{71}$$

where

$$\bar{\sigma}_{\ell 1} \sim -\frac{1}{8}\sqrt{\pi k R \sin\frac{\phi_1}{2}}\, \exp\left(-i2kR\sin\frac{\phi_1}{2}\right) i^{-\ell} k\left[(a_\ell - ib_\ell)\cdot\right.$$

$$\left.\cdot \exp\left\{i(\ell+1)\left(\frac{\phi_1}{2}+\frac{\pi}{2}\right)\right\} + (a_\ell + ib_\ell)\exp\left\{-i(\ell+1)\frac{\pi}{2}\right\}\right], \tag{72}$$

$$\bar{\sigma}_{\ell 2} \sim \frac{i\pi M(\ell-1)}{4\ell}\, e^{-i\ell\frac{\pi}{2}} \sum_{s=1}^{\infty} \frac{w_1(t_s) - w_2(t_s)}{w_1'(t_s)}\, k\left(a_\ell \sin\frac{\ell\pi}{2} + b_\ell \cos\frac{\ell\pi}{2}\right)\cdot$$

$$\cdot\left[\exp\left\{i\phi_1(kR+Mt_s)\right\} + \sum_{n=1}^{\infty}\left(\exp\left\{i(kR+Mt_s)(\phi_1+2\pi n)\right\}\right.\right.$$

$$\left.\left. + \exp\left\{-i(kR+Mt_s)(\phi_1-2\pi n)\right\}\right)\right] \tag{73}$$

In conclusion, the high frequency back scattered field due to a discontinuity in curvature is linearly proportional to the difference of the two radii of curvature and is of the order of: $k^{-9/6}$ if the discontinuity is in the illuminated region, $k^{-11/6}$ if it coincides with the shadow boundary, and $k^{-13/6}$ if it lies in the shadow region. If there is a discontinuity in the nth derivative of the curvature, the fields that have just been computed must be multiplied by a factor $O(k^{-n})$.

V
DIFFRACTION OF A DIPOLE FIELD BY A CONICAL RING

The electromagnetic field produced by the antenna of a space vehicle can be rigorously determined only for a few simple geometrical configurations of the system antenna-spacecraft. In the following, the diffracted electromagnetic field produced by an axially oriented electric dipole located on the axis of symmetry of a perfectly conducting conical ring is considered. This fairly exotic body occupies that region $\alpha < \theta < \pi - \alpha$ of a spherical shell $b < r < a$, which is limited by the surface of a cone of semi-angle $\theta = \alpha$. The free space surrounding the ring is divided into the regions I, II, III and IV of Fig. 4, and in each region the components of the diffracted field are derived from a properly chosen Hertzian function. The unknown coefficients which appear in the expressions of these Hertzian functions are then determined by requiring that the components of the total electromagnetic field satisfy the boundary conditions on the surface of the ring, and be continuous across the ideal surfaces which separate two adjacent regions of space.

Although it appears that this boundary value problem has not been previously considered, the method of solution, which makes use of Legendre functions of non-integral order and of an appropriate continuation technique, was first employed by Schelkunoff (1941) in his treatment of the biconical antenna, and subsequently by many other authors (e.g. Northover (1962) and Rogers, Schindler and Schultz (1963)).

With reference to a system of spherical polar coordinates (r, θ, ϕ), an axially oriented electric dipole is located on the axis of symmetry of the ring at a point $\theta = 0$, $r = h$. The components of the diffracted (incident plus scattered) electric field $\vec{E}$ and of the diffracted magnetic field $\vec{H}$ in the free space surrounding the ring can be derived from a scalar function $U(r, \theta)$ by means of the relations:

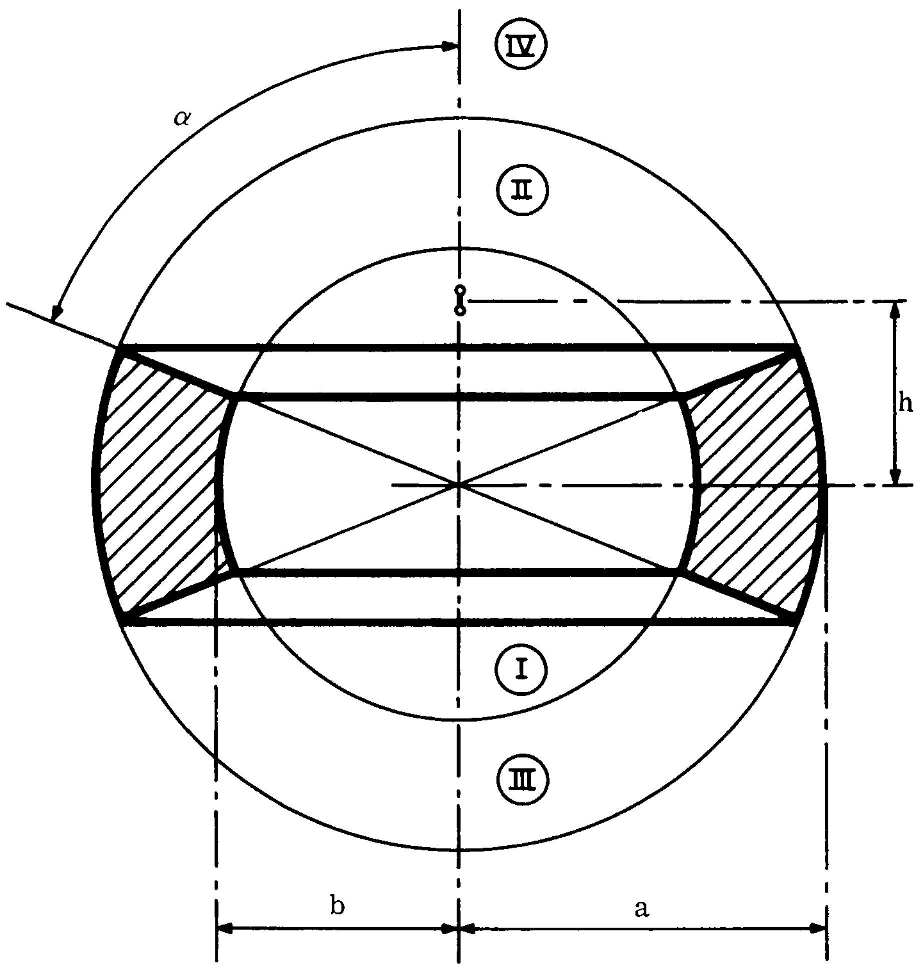

FIGURE 4

$$E_r = \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial U}{\partial\theta}\right), \tag{74}$$

$$E_\theta = -\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial U}{\partial\theta}\right) \quad , \tag{75}$$

$$H_\phi = -\frac{ik}{Z_o} \frac{\partial U}{\partial\theta} \quad , \tag{76}$$

$$E_\phi = H_r = H_\theta = 0 \ , \tag{77}$$

where $Z_o = 120\pi$ ohms is the free space intrinsic impedance. The function U must satisfy the reduced wave equation, the radiation condition, and the boundary conditions $U = 0$ on the slant surfaces $\theta = \alpha$ and $\theta = \pi - \alpha$ and $\frac{\partial(rU)}{\partial r} = 0$ on the spherical bases $r = a$ and $r = b$ of the conical ring. The primary field due to the electric dipole may be derived from the function

$$U_o = V \frac{e^{ikR}}{kR} \ , \tag{78}$$

where V is a constant with the dimensions of an electric voltage, and

$$R = (r^2 + h^2 - 2hr\eta)^{1/2} \quad , \qquad \eta = \cos\theta \ . \tag{79}$$

For the case $0 < h < b$, it is found that in region I ($r \leqslant b$):

$$U = U_1 = U_o + \frac{iV}{kh} \frac{1}{kr} \sum_{n=1}^{\infty} A_n \psi_n(kr)\, P_n(\eta) \quad , \tag{80}$$

in region II ($b \leqslant r \leqslant a;\ 0 \leqslant \theta < \alpha$):

$$U = U_2 = \frac{iV}{kh} \frac{1}{kr} \sum_{j=1}^{\infty} \left\{ C_j \psi_{\beta_j}(kr) + D_j \zeta_{\beta_j}(kr) \right\} P_{\beta_j}(\eta) \tag{81}$$

in region III ($b \leqslant r \leqslant a$; $\pi - \alpha < \theta \leqslant \pi$):

$$U = U_3 = \frac{iV}{kh}\frac{1}{kr}\sum_{j=1}^{\infty}\left\{\tilde{C}_j \psi_{\beta_j}(kr) + \tilde{D}_j \zeta_{\beta_j}(kr)\right\} P_{\beta_j}(-\eta) \ , \tag{82}$$

and in region IV ($r \geqslant a$):

$$U = U_4 = \frac{iV}{kh}\frac{1}{kr}\sum_{n=1}^{\infty} B_n \zeta_n(kr) P_n(\eta) \ , \tag{83}$$

where ψ_n and ζ_n are related to the Bessel and Hankel functions by the expressions

$$\psi_n(x) = \sqrt{\frac{\pi x}{2}}\, J_{n+1/2}(x), \qquad \zeta_n(x) = \sqrt{\frac{\pi x}{2}}\, H^{(1)}_{n+1/2}(x) \ , \tag{84}$$

and the positive numbers β_j are given by the equation:

$$P_{\beta_j}(\cos\alpha) = 0. \tag{85}$$

Let us set:

$$C_j^{\pm} = \frac{1}{2}(C_j \pm \tilde{C}_j) \ , \qquad D_j^{\pm} = \frac{1}{2}(D_j \pm \tilde{D}_j) \ , \tag{86}$$

then one finds that

$$A_n = -\frac{2n+1}{\psi_n'(kb)}\left[\psi_n(kh)\zeta_n'(kb) + \sum_{j=1}^{\infty} f_n(\beta_j)\left\{C_j^{\pm}\psi_{\beta_j}'(kb) + D_j^{\pm}\zeta_{\beta_j}'(kb)\right\}\right] , \tag{87}$$

$$B_n = -\frac{2n+1}{\zeta_n'(ka)}\sum_{j=1}^{\infty} f_n(\beta_j)\left\{C_j^{\pm}\psi_{\beta_j}'(ka) + D_j^{\pm}\zeta_{\beta_j}'(ka)\right\} \ , \tag{88}$$

where C_j^+ and D_j^+ are to be used for n even, C_j^- and D_j^- for n odd, and the primes indicate derivatives with respect to the argument ka or kb. The coefficients $C_j^\pm$ and $D_j^\pm$ are given by the two following infinite sets of linear algebraic equations in an infinite number of unknowns:

$$C_j^+ \psi_{\beta_j}(ka) + D_j^+ \zeta_{\beta_j}(ka) = - \sum_{m=1}^{\infty} (4m+1) \frac{f_{2m}(\beta_j)}{f(\beta_j)} \frac{\zeta_{2m}(ka)}{\zeta'_{2m}(ka)} \sum_{\ell=1}^{\infty} f_{2m}(\beta_\ell) \cdot$$

$$\cdot \left\{ C_\ell^+ \psi'_{\beta_\ell}(ka) + D_\ell^+ \zeta'_{\beta_\ell}(ka) \right\}, \tag{89a}$$

$$C_j^+ \psi_{\beta_j}(kb) + D_j^+ \zeta_{\beta_j}(kb) = - \sum_{m=1}^{\infty} \frac{4m+1}{\psi'_{2m}(kb)} \frac{f_{2m}(\beta_j)}{f(\beta_j)} \Bigg[i\psi_{2m}(kh) +$$

$$\psi_{2m}(kb) \sum_{\ell=1}^{\infty} f_{2m}(\beta_\ell) \left\{ C_\ell^+ \psi'_{\beta_\ell}(kb) + D_\ell^+ \zeta'_{\beta_\ell}(kb) \right\} \Bigg], \tag{89b}$$

$$C_j^- \psi_{\beta_j}(ka) + D_j^- \zeta_{\beta_j}(ka) = - \sum_{m=1}^{\infty} (4m-1) \frac{f_{2m-1}(\beta_j)}{f(\beta_j)} \frac{\zeta_{2m-1}(ka)}{\zeta'_{2m-1}(ka)} \cdot$$

$$\cdot \sum_{\ell=1}^{\infty} f_{2m-1}(\beta_\ell) \left\{ C_\ell^- \psi'_{\beta_\ell}(ka) + D_\ell^- \zeta'_{\beta_\ell}(ka) \right\}, \tag{90a}$$

$$C_j^- \psi_{\beta_j}(kb) + D_j^- \zeta_{\beta_j}(kb) = - \sum_{m=1}^{\infty} \frac{4m-1}{\psi'_{2m-1}(kb)} \frac{f_{2m-1}(\beta_j)}{f(\beta_j)} \Bigg[i\psi_{2m-1}(kh)$$

$$- \psi_{2m-1}(kb) \sum_{\ell=1}^{\infty} f_{2m-1}(\beta_\ell) \left\{ C_\ell^- \psi'_{\beta_\ell}(kb) + D_\ell^- \zeta'_{\beta_\ell}(kb) \right\} \Bigg] \tag{90b}$$

where:

$$f(\beta_j) = \int_1^{\cos\alpha} \left\{P_{\beta_j}(\eta)\right\}^2 d\eta \, , \tag{91}$$

$$f_n(\beta_j) = \int_1^{\cos\alpha} P_{\beta_j}(\eta)\, P_n(\eta)\, d\eta \quad . \tag{92}$$

Similar results are obtained in the case $h > a$. They are given in the original paper (Uslenghi,to be published, b) where many particular configurations of the ring are also examined in detail. The technique employed in this section can also be applied when $b < h < a$; the function U_o must then be expanded in terms of the functions $P_{\beta_j}(\eta)$, which are orthogonal in $0 \leqslant \theta \leqslant \alpha$.

The case in which the source is an axially oriented magnetic dipole located on the axis of symmetry of the conical ring is not very different from the electric dipole case, and is dealt with in a similar way. The diffracted field components can now be derived from a scalar function $W(r, \theta)$ through formulas:

$$H_r = \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial W}{\partial\theta}\right) \, , \tag{93}$$

$$H_\theta = -\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial W}{\partial\theta}\right) \, , \tag{94}$$

$$E_\phi = ikZ_o \frac{\partial W}{\partial\theta} \, , \tag{95}$$

$$H_\phi = E_r = E_\theta = 0 \, , \tag{96}$$

where $(\nabla^2 + k^2)W = 0$, and the boundary conditions reduce to $\frac{\partial W}{\partial\theta} = 0$ on the entire surface of the conical ring.

In general, the method of solution used in this section is applicable to all diffraction problems for which:

1) the scatterer is a perfectly conduction body of revolution, whose surface is made of portions of concentric spherical surfaces and of portions of conical surfaces having their common vertex at the center of the concentric spheres;

2) the source is an electric or magnetic dipole, axially oriented and located on the axis of symmetry of the scatterer.

The solutions presented in this section are of little practical utility as they stand, because the mode coefficients must satisfy infinite sets of linear algebraic equations that one is unable to solve. In any practical application, only a finite number of modes is taken into account, and numerical results are obtained with the aid of a computer. The simplest way of choosing these preferred modes is to consider only the first few terms (lower modes) of the infinite series which represent the Hertzian functions; this corresponds to replacing each infinite set of equations with a truncated set (an example of this procedure is given in Uslenghi, to be published, b). A better choice of the preferred set of modes might be based on the physical consideration that modes in two adjacent regions of space which are of like order will show maximum coupling to each other; this selective mode coupling was proposed and successfully employed by Plonus (1961, 1963) in his calculations of the radiation pattern of biconical antennas.

VI
OTHER THEORETICAL STUDIES

Some of the new results which were obtained in the course of the investigation and which are not included in the preceding sections are briefly described in the following. They include 1) a geometrical study of dielectric lenses, which originated in the attempt to obtain omnidirectionality through the use of a variable refractive index layer within which a source is placed, 2) a study of the high frequency back scattering from a coated cylinder, and 3) a similar study for a coated sphere.

Let us consider dielectric lenses with spherical or cylindrical symmetry, in which the index of refraction N is a function of the distance r from the center of the sphere or from the axis of the cylinder. Assume that the radius of the sphere, or of the cylinder, is equal to unity, that $N(r)$ is finite except possibly at $r = 0$, that $N(1) = 1$, and that $N(r)$ is continuous in $0 < r < 1$. The geometry is the same for both spherical and cylindrical lenses, and is illustrated in Fig. 5. An optical ray

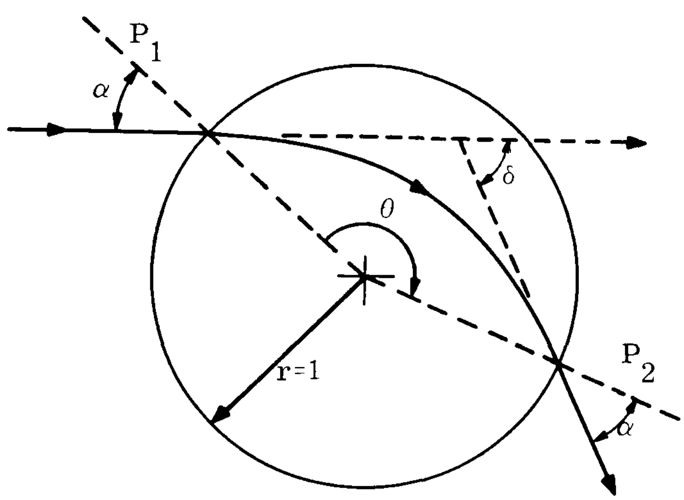

FIGURE 5

enters the lens at P_1 with an angle of incidence α, it is smoothly deviated along the path P_1P_2, and leaves the lens at P_2. Following the usual procedure, let us set

$$r^2N^2 = 1 - \xi^2 , \tag{97}$$

and introduce a new function $f(\xi)$ satisfying the relation:

$$2N^2 r\,dr = -f(\xi)d\xi . \tag{98}$$

If $f(\xi)$ is an analytic function of ξ in a neighborhood of $\xi = 0$, then restrictions have to be placed on the coefficients of the power series representing $f(\xi)$, if one wants to have solutions with physical significance. Let

$$f(\xi) = \sum_{n=0}^{\infty} a_n \xi^n , \tag{99}$$

then the coefficients a_n cannot be arbitrarily chosen; in fact, the angular extension θ of the ray path P_1P_2 as seen from the center $r = 0$ must obviously be positive for the entire range $0 \leqslant \alpha < \pi/2$. From the relation

$$\theta = \sin\alpha \int_0^{\cos\alpha} \frac{f(\xi)\,d\xi}{(1-\xi^2)\sqrt{\cos^2\alpha - \xi^2}} , \tag{100}$$

one sees that, if $f(\xi)$ is a polynomial of degree n in ξ, then the fundamental limitation on the choice of the coefficients is:

$$\theta(a_o, a_1, \ldots, a_n; \alpha) > 0, \qquad \text{for all } 0 \leqslant \alpha < \pi/2, \tag{101}$$

i.e. the representative point $(a_o, a_1, \ldots, a_n)$ must belong to a certain region of the $(n+1)$-dimensional space $(a_o, a_1, \ldots, a_n)$. From (97) and (98) it is seen that the relationship between N and r remains unchanged when both ξ and $f(\xi)$ change sign, i.e. when ξ and all the coefficients a_{2m} change sign. The ambiguity that thus arises in the choice of the representative point $(a_o, a_1, \ldots)$ for a given lens $N = N(r)$ is eliminated by requiring that inequality (101) be satisfied. This inequality has previously been used by Huynen (1958) in the particular case of the Luneberg lens.

As an example, let us consider the simple but important case

$$f(\xi) = a_o + a_1 \xi \ , \tag{102}$$

for which relation (100) reduces to

$$\theta = \frac{\pi}{2} a_o + \left(\frac{\pi}{2} - \alpha\right) a_1 ; \tag{103}$$

then relation (101) gives:

$$a_o + a_1 > 0, \qquad a_o \geqslant 0 \ , \tag{104}$$

so that a_o and a_1 cannot be chosen arbitrarily. More details are given in the original paper (Uslenghi, 1964c).

The determination of the high frequency radar cross section of a smooth convex conducting body covered with one or more thin absorbing layers of materials with large complex indexes of refraction (e.g. ferrites) can be greatly simplified by observing that under certain general assumptions the total electric and magnetic fields satisfy an impedance boundary condition on the outer surface of the outer coating layer. In certain practical applications, such a scatterer is in turn covered by another layer of material whose index of refraction is no longer large compared to unity. It is then of great practical importance to investigate the influence that this outer layer has on the magnitude of the far back scattered field, and therefore on the value of the monostatic radar cross section. The analysis is complicated by the fact that the exact boundary conditions must be imposed at the outer surface of the outer layer, while an impedance boundary condition may still be assumed on its inner surface, provided that the index of refraction of the material of the inner layer is very large and has a large imaginary part, and that the radii of curvature of the outer surface of the inner layer are large compared to the wavelength inside the layer itself.

In the following, the investigation is carried out for the case of an infinitely long circular coated cylinder. It is supposed that the material of the outer coating layer has a complex refractive index whose absolute value has a lower bound that is only moderately large compared to unity (e.g. 1.5 or 2), and whose argument is bounded away from both zero and $\pi/2$. An asymptotic evaluation of the far back scattered field is obtained in terms of the geometric optics and of the creeping wave contributions, for small wavelengths and normal incidence.

Consider an infinitely long cylinder of radius b, coated with a layer of constant thickness d and surrounded by free space; the radius a of the outer surface is then equal to $(b+d)$. Let ϵ and μ be the relative permittivity and permeability of the material of the layer, and let η be the relative surface impedance at $r=b$. The wave number k_1 of the coating is related to the index of refraction $N=\sqrt{\epsilon\mu}$ by the expression $k_1=Nk$. The incident wave $E_z^{(i)} = -\sqrt{\mu_o/\epsilon_o}\,H_y^{(i)} = \exp(ikx)$ is propagating in the positive x direction, perpendicularly to the axis z of the cylinder, and is polarized in the (x, z) plane. The far back scattered field is given by:

$$E_z^{(b.s.)} \sim \sqrt{\frac{2}{\pi kr}}\; e^{ikr - i\frac{\pi}{4}} \left[a_o + 2\sum_{n=1}^{\infty} (-1)^n a_n \right], \tag{105}$$

where:

$$a_n = -\frac{J_n'(ka) - A_n J_n(ka)}{H_n^{(1)'}(ka) - A_n H_n^{(1)}(ka)}, \tag{106}$$

with

$$A_n = \frac{N}{\mu}\,\frac{\partial(\ln C_n)}{\partial(k_1 a)}\;\frac{1 - i\eta\,\dfrac{N}{\mu}\,\dfrac{\partial}{\partial(k_1 b)}\left[\ln \dfrac{\partial C_n}{\partial(k_1 a)}\right]}{1 - i\eta\,\dfrac{N}{\mu}\,\dfrac{\partial(\ln C_n)}{\partial(k_1 b)}} \tag{107}$$

and

$$C_n = J_n(k_1 a)\, H_n^{(1)}(k_1 b) - J_n(k_1 b)\, H_n^{(1)}(k_1 a) \ . \tag{108}$$

It is assumed that

$$\Phi < \arg N < \frac{\pi}{2} - \Phi' \tag{109}$$

with

$$\Phi \gg |k_1 b|^{-2/3} \ , \qquad \Phi' \gg |k_1 b|^{-1} \tag{110}$$

and that $|k_1 d|$ is not large compared to unity. By applying a Watson transform technique, it is found that the geometric optics contribution to the back scattered field for large ka is given by:

$$\left[E_z^{(b.s.)}\right]_{g.o.} \sim \frac{\tilde{\eta}-1}{\tilde{\eta}+1}\sqrt{\frac{a}{2r}}\, e^{ikr - i2ka}\left[1 + \frac{i}{2ka}\left\{\frac{5}{8} + \frac{\tilde{\eta}}{\tilde{\eta}^2 - 1}\left[1 - 2\tilde{\eta}\left(1 + \frac{ip}{N^2}\right)\right]\right\} + \ldots\right], \tag{111}$$

where:

$$\tilde{\eta} = -i\,\frac{\mu}{N}\tan\left[k_1 d + \arctan\left(i\eta\frac{N}{\mu}\right)\right] \ , \tag{112}$$

$$p = \frac{\frac{N}{\mu}\cot k_1 d}{1 + i\eta\frac{N}{\mu}\cot k_1 d}\left[\frac{1}{2}\left\{1 + \left(\eta\frac{N}{\mu}\right)^2\right\} - \frac{k_1 d}{\sin 2k_1 d}\left\{1 - \left(\eta\frac{N}{\mu}\right)^2\right\} - i\eta\frac{N}{\mu}\tan k_1 d\right] \ . \tag{113}$$

The creeping wave contribution is given by the residue series:

$$\left[E_z^{(b.s.)}\right]_{cr.w.} \sim \frac{2\sqrt{2\pi}}{m}\, e^{-i\frac{3\pi}{4}} \frac{e^{ikr}}{\sqrt{kr}} \sum_s \left[\sin(\pi\nu_s)\, w_1^2(t_s)\, \left\{\eta_1^{-2} + \frac{p_1}{m^3} + \frac{t_s}{m^2}\left(1 + i\,\frac{2p_1}{\eta_1}\right)\right\}\right]^{-1} , \tag{114}$$

where:

$$\nu_s = ka + mt_s , \qquad m = (ka/2)^{1/3} \tag{115}$$

t_s are the roots of

$$\frac{w_1'(t)}{w_1(t)} = \frac{im}{\eta_1} - p_1 \frac{t}{m} , \tag{116}$$

and

$$\eta_1 = \frac{-i\mu}{\sqrt{N^2-1}} \tan\left[kd\sqrt{N^2-1} + \arctan\left(i\eta\,\frac{\sqrt{N^2-1}}{\mu}\right)\right], \tag{117}$$

$$p_1 = \frac{1}{2\mu\sqrt{N^2-1}} \left(1 + i\eta\,\frac{\beta\cot\beta}{\mu kd}\right)^{-2} \left[\frac{\beta}{\sin^2\beta} - \cot\beta + i\,\frac{2\eta\beta}{\mu kd}\left(1 + i\eta\,\frac{\beta\cot\beta}{\mu kd}\right)\right], \tag{118}$$

$$\beta = kd\sqrt{N^2-1} \tag{119}$$

The result (114) is valid if ka is large and if:

$$\left|\nu_s - k_1 a\right| > \left|\nu_s\right|^{1/3} , \qquad \left|\nu_s - k_1 b\right| > \left|\nu_s\right|^{1/3} , \tag{120}$$

$$\left|t_s\right| \ll m^2 , \qquad \left|\frac{kd}{2\sqrt{N^2-1}}\, t_s/m^2\right| \ll 1 . \tag{121}$$

The total back scattered field is obtained by adding together the contributions (111) and (114). For details on the derivations and for some numerical results, the reader is referred to the original paper (Uslenghi, 1964b).

A similar study was carried out for the case of a large sphere of radius ρ and relative surface impedance $\eta\sqrt{\epsilon_o/\mu_o}$, coated with two concentric layers of different materials and surrounded by free space (Uslenghi, 1964, to be published, a). Since the results are even more complicated that in the cylindrical case, they will not be given here.

REFERENCES

Barlow, H.M. (1959) IRE Trans. AP-7, S147-S153.

Clemmow, P.C. and V.H. Weston (1961) Proc. Roy. Soc., A 264, 246-268.

Garabedian, P.R. (1955) Quart. Appl. Math., 12, 428-433.

Huynen, J.R. (1958) IRE Wescon Cenvention Record, Part I, 219-230.

Keller, J.B. (1962) "Diffraction by Polygonal Cylinders", from Electromagnetic Waves, ed. R.E. Langer (The University of Wisconsin Press, Madison) 129-137.

Northover, F.H. (1962) Quart. J. Mech. Appl. Math., 15, part 1, 1-9.

Plonus, M.A. (1961) "A Study of the Biconical Antenna", Radiation Laboratory Report No. 3620-1-F.

Plonus, M.A. (1963) "Application of Selective Mode Coupling in the Solution to Biconical Antennas", from Electromagnetic Theory and Antennas, Part 2 , Ed. E.C. Jordan (Pergamon Press, Oxford) 1155-1166.

Rhodes, D.R. (1963) IEEE Trans., AP-11, 440-446.

Rogers, CCC., J.K. Schindler and F.V. Schultz (1963) "The Scattering of a Plane Electromagnetic Wave by a Finite Cone", from Electromagnetic Theory and Antennas, Part 1 , ed. E.C. Jordan (Pergamon Press, Oxford) 67-80.

Senior, T.B.A. (1964) "Extent to which the Surface Field is Specified by a Far-Field Power Polar Diagram", Radiation Laboratory Memorandum No. 5780-512-M.

Uslenghi, P.L.E. (1963) "The Optimum Approximation to a Given Radiation Pattern for Strip Source Antennas on Imperfectly Conducting Cylinders", Radiation Laboratory Memorandum No. 5780-502-M.

Uslenghi, P.L.E. (1964a) "On Experimental Investigation on Omnidirectional Antennas", Radiation Laboratory Memorandum No. 5780-509-M.

Uslenghi, P.L.E.,(1964b) "High Frequency Scattering from a Coated Cylinder", (to appear in November issue of Canadian Journal of Physics).

Uslenghi, P.L.E. (1964c) "A Note on Dielectric Lenses" (to appear in November issue of Canadian Journal of Physics).

Uslenghi, P.L.E. (To be published, a) "High Frequency Back Scattering from a Coated Sphere".

Uslenghi, P.L.E. (To be published, b) "Diffraction of a Dipole Field by a Conical Ring".

Warshawski, S. (1935) Trans. Amer. Math. Soc., 38, 310-340.

Weston, V.H. (1962) IEEE Trans., AP-10, 775-780.

APPENDIX
BIBLIOGRAPHY OF PUBLICATIONS UNDER THIS GRANT

Introduction

Hiatt, R. E., "Study of Antenna Problems Associated with NASA Space Vehicles", Proposal No. ORA-63-522-PB1 (December, 1962).

Senior, T. B. A., "First Semi-Annual Progress Report", Radiation Laboratory Memorandum No. 5780-1-T (20 November 1963).

Senior, T. B. A., "Second Semi-Annual Progress Report", Radiation Laboratory Memorandum No. 5780-2-T (July, 1964).

Section II

Hiatt, R. E., "Experimental Work on Omnidirectional Antennas", Radiation Laboratory Memorandum No. 5780-503-M (5 November 1963).

Uslenghi, P. L. E., "On Experimental Investigation on Omnidirection Antennas", Radiation Laboratory Memorandum No. 5780-509-M, (17 March 1964).

Uslenghi, P. L. E., "On the Circumferential TM Surface Wave Supported by a Coated Cylinder", Radiation Laboratory Memorandum 5780-504-M (19 November 1963).

Uslenghi, P. L. E., "On the Circumferential TM Surface Wave Supported by a Coated Cylinder - Part 2", Radiation Laboratory Memorandum No. 5780-505-M (14 January 1964).

Section III

Senior, T. B. A., "Extent to Which the Surface Field is Specified by a Far-Field Power Polar Diagram", Radiation Laboratory Memorandum No. 5780-512-M (22 May 1964).

Uslenghi, P. L. E., "The Optimum Approximation to a Given Radiation Pattern for a Circumferential Slot on a Metal Cylinder", Radiation Laboratory Memorandum No. 5780-510-M (11 May 1964).

Uslenghi, P. L. E., "The Optimum Approximation to a Given Radiation Pattern for Strip Source Antennas on Imperfectly Conduction Cylinders", Radiation Laboratory Memorandum No. 5780-502-M (16 October 1963).

Section IV

Hong, S., "First Order Theory of Scsttering by a Cylinder with Piecewise Smooth Surface", Radiation Laboratory Memorandum No. 5780-501-M (31 July 1963).

Hong, S., "Addenda to Memo of 31 July 1963", Radiation Laboratory Memorandum No. 5780-501a-M (20 August 1963).

Hong, S., "Reformulation of the Integral Representation of Two Dimensional Electromagnetic Waves and General Discussion of this Method, Radiation Laboratory Memorandum No. 5780-506-M (4 December 1963).

Hong, S., "High Frequency Scattering by a Cylinder with Discontinuous Curvature - An Ogive", Radiation Laboratory Memorandum No. 5780-507-M (7 February 1964).

Hong, S., "Application of Conformal Mapping to Radiation and Diffraction of Electromagnetic Waves", Radiation Laboratory Paper No. RL-254 (25 February 1964).

Hong, S., "Study of the Effect of the Non-Analiticity of the Boundary in High Frequency Scattering", Radiation Laboratory Memorandum No. 5780-511-M (19 May 1964).

Section V

Uslenghi, P. L. E., "Diffraction of a Dipole Field by a Conical Ring" (submitted for publication in Applied Scientific Research), September, 1964.

Section VI

Uslenghi, P. L. E., "High Frequency Scattering from a Coated Cylinder", (to appear in the November 1964 issue of the Canadian Journal of Physics)

Uslenghi, P. L. E., "High Frequency Back Scattering from a Coated Sphere" (submitted for publication in Alta Frequenza), May, 1964.

Uslenghi, P. L. E., "A Note on Dielectric Lenses", (to appear in the November 1964 issue of the Canadian Journal of Physics).

Uslenghi, P. L. E., "A Study of Dielectric Lenses", (presented at the URSI-IEEE Fall Meeting at Urbana, Illinois, 13 October 1964).

NASA-Langley, 1965 CR-159